Commun. Math. Phys. 158, 127–134 (1993)

Communications in
Mathematical
Physics
© Springer-Verlag 1993

On Two Integrable Cellular Automata**

Alexander Bobenko[1,*], **Martin Bordemann**[2], **Charlie Gunn**[1], **Ulrich Pinkall**[1]

[1] Fachbereich Mathematik, Technische Universität Berlin, Strasse des 17. Juni 136, D-10623 Berlin, Germany
[2] Fachbereich Physik der Universität Freiburg, Hermann-Herder-Str. 3, D-79104 Freiburg, Germany

Received: 16 July 1992/in revised form: 26 February 1993

Abstract. We describe two simple cellular automata (CA) models which exhibit the essential attributes of soliton systems. The first one is an invertible, 2-state, 1-dimensional CA or, in other words, a nonlinear $\mathbf{Z}_2$-valued dynamical system with discrete space and time. Against a vacuum state of 0, the system exhibits light cone particles in both spatial directions, which interact in a soliton-like fashion. A complete solution of this system is obtained. We also consider another CA, which is described by the Hirota equation over a finite field, and present a Lax representation for it.

1. Introduction

Cellular automata (CA) have become increasingly popular models for physical systems [9]. CA are regular grids of finite state automata each of whose states at successive time steps is determined uniformly by the states of some finite neighborhood. In the simplest case the grid is a one-dimensional array. This dimension can be referred to as the x-dimension and CA can be described as a dynamical system in discret space and time whose field variables take only finitely many values. CA were found with coherent particle-like structures. Some of these particles scatter as solitons [7, 1].

We suggest new time-reversable CA, which we call soliton systems, because they possess the usual features of the integrable systems. Let us consider a diagonally oriented lattice and some horizontal stairway S on it (shaded in Fig. 1).

We consider a field v taking values in some finite set $\mathscr{F}$ at each face of S and which obey an evolution equation of the form

$$v_d = F(v_l, v_u, v_r)\,.$$

(We direct time down)

* On leave of absence from St. Petersburg Branch of Steklov Mathematical Institute, St. Petersburg, Russia

** Supported by Deutsche Forschungsgemeinschaft, Sonderforschungsbereich 288