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A One-Parameter Family of Hamiltonian Structures for the KP Hierarchy and a Continuous Deformation of the Nonlinear W_{KP} Algebra

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Abstract: The KP hierarchy is hamiltonian relative to a one-parameter family of Poisson structures obtained from a generalized Adler map in the space of formal pseudodifferential symbols with noninteger powers. The resulting W-algebra is a one-parameter deformation of W_{KP} admitting a central extension for generic values of the parameter, reducing naturally to W_n for special values of the parameter, and contracting to the centrally extended $W_{1+\infty}$, W_∞ and further truncations. In the classical limit, all algebras in the one-parameter family are equivalent and isomorphic to w_{KP}. The reduction induced by setting the spin-one field to zero yields a one-parameter deformation of $\hat{W}_\infty$ which contracts to a new nonlinear algebra of the W_∞-type.

1. Introduction

The topography of W-algebras [1] in two dimensions is beginning to unfold and, among them, algebras of the W_∞-type provide natural landmarks. Some of these W-algebras, which are generated by fields of integer weights 2, 3, 4, . . . and possibly also 1, are expected to be universal for some infinite series of finitely generated W-algebras, in the sense [2] that all W-algebras in that series can be obtained from it as reductions. The best-known example of such a series is comprised by the W_n algebras [3], of which the Virasoro algebra (corresponding to $n = 2$) is the simplest.

These algebras can be realized classically (i.e., as Poisson algebras) as a certain natural reduction of the second Gel'fand-Dickey brackets [4] – a hamiltonian structure for the generalized KdV hierarchies (see [5] for a comprehensive review). These are the integrable hierarchies of isospectral deformations (of Lax type) of the one-dimensional differential operator $L = \partial^n + \sum_{j=0}^{n-1} u_j(z)\partial^j$ in terms of which, the Gel'fand–Dickey brackets have a very simple expression which we now briefly review.

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