

Quantum Algebra Structure of Certain Jackson Integrals

Atsushi Matsuo

Department of Mathematics, Nagoya University, Nagoya 464-01, Japan

Received: June 29, 1992; in revised form February 19, 1993

Abstract. The q -difference system satisfied by Jackson integrals with a configuration of A -type root system is studied. We explicitly construct some linear combination of Jackson integrals, which satisfies the quantum Knizhnik–Zamolodchikov equation for the 2-point correlation function of q -vertex operators, introduced by Frenkel and Reshetikhin, for the quantum affine algebra $U_q(\widehat{\mathfrak{sl}}_2)$. The expression of integrands for the n -point case is conjectured, and a set of linear relations for the corresponding Jackson integrals is proved.

1. Introduction

1.1. In a recent work [FR], Frenkel and Reshetikhin constructed a q -analogue of the vertex operators of Tsuchiya and Kanie [TK] and derived a q -difference equation for the n -point correlation function, which is a q -analogue of the Knizhnik–Zamolodchikov equation (KZ) in the Wess–Zumino–Witten model and is called the quantum Knizhnik–Zamolodchikov equation (qKZ). This equation is written in terms of the trigonometric R-matrices arising from finite dimensional representations of the quantum affine algebra $U_q(\hat{\mathfrak{g}})$ (cf. [J1]) corresponding to a simple Lie algebra \mathfrak{g} , and plays an essential role to produce elliptic solutions of the quantum Yang–Baxter equation (YBE) of IRF type. One needs to solve the connection problem of qKZ to obtain an explicit form of the elliptic solutions of YBE in this context.

Therefore to indicate solutions of qKZ explicitly is of great importance to consider the following question: Do all the known elliptic solutions of YBE of IRF type come from the connection matrix of qKZ? Actually in the simplest case for $U_q(\widehat{\mathfrak{sl}}_2)$, Heine's basic hypergeometric functions with Jackson integral representations are used to represent solutions to qKZ for 2-point function in [FR] and the resulting connection matrix coincides with the ABF-solution of YBE (cf. [ABF, JMO]).

In a previous paper [Ma2], we have constructed a Jackson integral solution to qKZ for the n -point function of the q -vertex operators of $U_q(\widehat{\mathfrak{sl}}_2)$ for certain kinds