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A Quiver Quantum Group

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Abstract: We construct quantum groups at a root of unity and we describe their monoidal module category using techniques from the representation theory of finite dimensional associative algebras.

1. Introduction

In the representation theory of finite dimensional algebras, important techniques as quivers, almost-split sequences and irreducible morphisms have been developed these last two decades. Our purpose is to use these methods in order to study finite dimensional Hopf algebras which are neither commutative nor cocommutative together with their monoidal category of modules. Hopf algebras of this sort produce non-trivial braided categories through the representations of the quasi-triangular Drinfeld's double ([8]) or through the "center construction" ([10, 13, 18, 15]); according to Drinfeld [8] or Manin [19] these Hopf algebras are considered as quantum groups or rather better as functions on some hypothetic quantum group. Their representation theory through the double constructions has applications in various parts of physics and mathematics: one can use them to obtain solutions of the quantum Yang–Baxter equation or topological invariants (see for instance [24, 25, 17]).

Quivers with relations allow to present finite dimensional associative algebras in a useful way, see [11]. If the algebra is of finite representation type, its module category can be presented using the Auslander–Reiten quiver of the category of indecomposable modules and relations given by the almost-split sequences.

In this note we construct a family of infinite dimensional Hopf algebras over an arbitrary field k. To each integer $n \geqq 2$ and each n^{th} root of unity q in k we attach a Hopf algebra $H_n(q)$ which is the path algebra of the cyclic quiver Z_n. These Hopf

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