Commun. Math. Phys. 156, 399–433 (1993)

Communications in Mathematical Physics

Spectral Gap and Logarithmic Sobolev Inequality for Kawasaki and Glauber Dynamics

Sheng Lin Lu[1], **Horng-Tzer Yau**[2,⋆]

[1] Department of Mathematics, University of Michigan, Ann Arbor, MI 48109-1092, USA
e-mail: shenglu@math.lsa.umich.edu
[2] Courant Institute, New York University, New York, NY 10012, USA
e-mail: yau@math.nyu.edu

Received: 29 December 1992; in revised form: 19 February 1993

Abstract. We prove that the spectral gap of the Kawasaki dynamics shrink at the rate of $1/L^2$ for cubes of size L provided that some mixing conditions are satisfied. We also prove that the logarithmic Sobolev inequality for the Glauber dynamics in standard cubes holds uniformly in the size of the cube if the Dobrushin-Shlosman mixing condition holds for standard cubes.

Introduction

As the simplest model in statistical mechanics, Ising model has been studied extensively. It is by far the most studied model in mathematical physics and its phase structures were analyzed in great detail. The study of dynamical properties of the Ising model, on the other hand, is in a much more primitive stage. Our main concern is the hydrodynamical limit of the Ising model for which we shall provide a basic estimate on the gap of Kawasaki dynamics. The hydrodynamical limit of various models has been studied recently and several useful methods were developed, see, e.g. [DP, S] for a review. A central assumption of these methods is the so-called *gradient condition*. Roughly speaking, it means that the current of the dynamics is by itself a gradient of some other quantity. For models with this property, a natural summation by parts can be performed and the technical difficulty is greatly reduced. The drawback of gradient models is that the diffusion coefficient, as given by the Green-Kubo formula, is determined by the thermodynamical quantities rather than depending on correlation functions as the nongradient model does. Therefore, it does not manifest effects of fluctuations on the diffusion coefficient.

Another interesting aspect of the gradient condition is that, except in dimension $d = 1$ or the infinite temperature case, no gradient model has been constructed for any truly interacting, reversible models with discrete spin space. So a study of the

⋆ Research partially supported by U.S. National Science Foundation grant 9101196, Sloan Foundation Fellowship and David and Lucile Packard Foundation Fellowship