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Mean Field Theory of Directed Polymers with Random Complex Weights

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Abstract. We show that for the problem of directed polymers on a tree with i.i.d. random complex weights on each bond, three possible phases can exist; the phase of a particular system is determined by the distribution ρ of the random weights. For each of these three phases, we give the expression of the free energy per unit length in the limit of infinitely long polymers. Our proofs require several hypotheses on the distribution ρ, most importantly, that the amplitude and the phase of each complex weight be statistically independent. The main steps of our proofs use bounds on noninteger moments of the partition function and self averaging properties of the free energy. We illustrate our results by some examples and discuss possible generalizations to a larger class of distributions, to Random Energy Models, and to the finite dimensional case. We note that our results are not in agreement with the predictions of a recent replica approach to a similar problem.

1. Introduction

The behavior of directed polymers in a random medium has become over the last few years a central problem in the theory of disordered systems [10, 13, 17, 18, 19, 22, 25, 27]. In its lattice version the problem can be formulated as follows: a random energy is assigned to each bond of the lattice, and every directed walk on the lattice has an energy given by the sum of all the bonds visited by the walk. As usual in statistical mechanics, the problem is to understand the thermal equilibrium of this system; in particular, we wish to calculate its partition function.

The problem is related to several physical phenomena: interfaces in two dimensional disordered magnets [14], the pinning of vortex lines by impurities, and the growth of the surfaces of Eden clusters and of ballistic deposits [18, 20]. The problem also has many features in common with spin glasses, particularly at the mean field level [10]. (There are traditionally several ways of defining the mean field theory of a system in statistical mechanics. Here, as in [10], we will use this term to refer to the model of directed polymers in which the lattice is taken to be the Cayley tree.) This mean field case is so far one of the very few disordered systems for which it has been possible to prove that the predictions of the replica theory, in