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Steady-State Electrical Conduction in the Periodic Lorentz Gas

N.I. Chernov[1,2], **G.L. Eyink**[3], **J.L. Lebowitz**[3], **and Ya.G. Sinai**[4]

[1] Center for Dynamical Systems and Nonlinear Studies, Georgia Institute of Technology, Atlanta, Ga 30332, USA
[2] Joint Institute for Nuclear Research, Dubna, P.O. Box 79, Head Post Office, Moscow, Russia
[3] Departments of Mathematics and Physics, Rutgers University, New Brunswick, NJ 08903, USA
[4] L.D. Landau Institute for Theoretical Physics, Russian Academy of Sciences, 117940 GSP-1 Moscow V-334, Russia

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Abstract. We study nonequilibrium steady states in the Lorentz gas of periodic scatterers when an electric external field is applied and the particle kinetic energy is held fixed by a "thermostat" constructed according to Gauss' principle of least constraint (a model problem previously studied numerically by Moran and Hoover). The resulting dynamics is reversible and deterministic, but does not preserve Liouville measure. For a sufficiently small field, we prove the following results: (1) existence of a unique stationary, ergodic measure obtained by forward evolution of initial absolutely continuous distributions, for which the Pesin entropy formula and Young's expression for the fractal dimension are valid; (2) exact identity of the steady-state thermodynamic entropy production, the asymptotic decay of the Gibbs entropy for the time-evolved distribution, and minus the sum of the Lyapunov exponents; (3) an explicit expression for the full nonlinear current response (Kawasaki formula); and (4) validity of linear response theory and Ohm's transport law, including the Einstein relation between conductivity and diffusion matrices. Results (2) and (4) yield also a direct relation between Lyapunov exponents and zero-field transport (= diffusion) coefficients. Although we restrict ourselves here to dimension $d = 2$, the results carry over to higher dimensions and to some other physical situations: e.g. with additional external magnetic fields. The proofs use a well-developed theory of small perturbations of hyperbolic dynamical systems and the method of Markov sieves, an approximation of Markov partitions.

I. Physical Discussion and Statement of Results

(a) *Introduction.* We consider in this paper a dynamical system which corresponds to the motion of a single particle between a finite number of fixed, disjoint, convex scatterers in a periodic domain of the plane $\mathbb{R}^2$. As in the previous works [2, 3], the particle changes its velocity at moments of collision according to the usual law of elastic reflection, but, unlike there, the particle motion between collisions is not the free one at constant velocity. Instead, the motion between collisions is governed by