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Representations of Central Extensions of Differentiably Simple Lie Superalgebras

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Abstract. Let S be a simple Lie algebra of characteristic 0 and $\Lambda(n)$ be the Grassmann superalgebra in n indeterminates. We can form the Lie superalgebra $S \otimes \Lambda(n)$. The purpose of this paper is to classify all finite dimensional irreducible representations of all central extensions of $S \otimes \Lambda(n)$. We will also give a character formula for these representations.

0. Introduction

A finite dimensional differentiably simple Lie superalgebra is a Lie superalgebra that has no nontrivial differential ideals. In [K1] Kač suggested using methods developed in [B] to show that every such Lie superalgebra is the tensor product of a simple Lie superalgebra and a Grassmann superalgebra. It is not very hard to show that their finite dimensional irreducible representations are just irreducible representations of the simple Lie superalgebra with the unique maximal ideal acting trivially. A proof of this can be found in [C]. In this paper we will study central extensions of the Lie superalgebra $S \otimes \Lambda(n)$, where S is a simple Lie algebra. We will explicitly construct central extensions and use this to obtain a character formula for finite dimensional irreducible representations of such extensions.

We call the tensor product of the Laurent polynomials (in several, say m, variables) and a Grassmann superalgebra the superalgebra of *super Laurent polynomials*. Now let S be a finite dimensional simple Lie superalgebra. The general loop superalgebra associated to S is obtained by taking the tensor product of S and the super Laurent polynomials. Following the definition of the current (= affine Kač–Moody algebra) algebra, we call the central extensions of the general loop superalgebra the *general current superalgebra*. This paper deals with the special case when S is a Lie algebra and $m = 0$. It is our hope that it can shed some light on the representation theory of the general case.

This paper is organized as follows: In Sect. 1 we prove a 1–1 correspondence between central extensions and "superskewsymmetric derivations." To do this we apply a technique used in the construction of the current algebra starting with the loop algebra. This method is due to Kač, and sketches of it can be found in some of