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Harmonization and Homogenization on Fractals

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Abstract. This paper suggests a direct approach to define the Laplacian, the spectral dimension of nested fractals and the pre-Sierpinski carpet conductivity. We find a geometric construction of the harmonic functions on the gasket and therefore can describe effectively the dense set of functions having finite energy. The paper is mostly aimed at the homogenization on the pre-Sierpinski gasket, whose horizontal and nonhorizontal bonds have different conductivities: a and b respectively. We prove the Γ-convergence of the rescaled energies on the pre-Sierpinski gasket to $\sigma(a, b)\varepsilon$, where ε is the standard energy on the gasket with uniform conductivities. We also find an explicit expression for the effective conductivity $\sigma(a, b)$ and deduce that its set of singularities turns out to be the Julia set of a certain rational function. A special section is devoted to the problem of the pre-Sierpinski carpet conductivity asymptotic behavior; for this problem a new proof of Barlow-Bass inequalities with sharper constants is given.

1. Introduction

The fractals, the first example of which was given by Sierpinski [1] at the beginning of the century as an example of the set with the bizzare geometrical properties, were proposed more recently as models for different physical phenomena by Mandelbrot [2]. Then the Laplacian on the fractals and their spectral dimension which first appeared in the physical literature [2, 3], see review [4]) as the tools of the investigation of the percolation effects and various transport processes, in classical as well as in quantum mechanics became the subject of intensive mathematical research [5–9]. Even in the case of fractals with uniform properties, and all the quoted papers devoted to that case, this subject is related to the theory of certain inhomogeneous media and has something in common with homogenization theory. At the same time the main assumption of that theory (which is in the most general case statistical translation invariance) is violated in the fractal case. In this paper we go further and, probably, for the first time, at least in the mathematical literature,