

## Perturbative Renormalization of Composite Operators via Flow Equations II: Short Distance Expansion

## G. Keller<sup>1</sup>\* and C. Kopper<sup>2</sup>

<sup>1</sup> Max-Planck-Institut für Physik, Werner-Heisenberg-Institut, Föhringer Ring 6, W-8000 München 40, Germany

 $^2$ Institut für Theoretische Physik, Universität Göttingen, Bunsenstrasse 9, W-3400 Göttingen, Germany

Received May 21, 1992

Abstract. We give a rigorous and very detailed derivation of the short distance expansion for a product of two arbitrary composite operators in the framework of the perturbative Euclidean massive  $\Phi_4^4$ . The technically almost trivial proof rests on an extension of the differential flow equation method to Green functions with bilocal insertions, for which we also establish a set of generalized Zimmermann identities and Lowenstein rules.

## **Table of Contents**

- 1. Introduction
- 2. General Setting

2.1. Definition of the bare interaction; 2.2. The differential flow equation; 2.3. The boundary conditions at  $\Lambda = \Lambda_0(D^{(1,2)} \ge 0)$ ; 2.4. The renormalization conditions  $(D^{(1,2)} \ge 0)$ ; 2.5. Perturbative renormalizability

- Normal Products and Some (Generalized) Zimmermann Identities
  Generating functionals and their flow equations; 3.2. Definition of bilocal normal products; 3.3. Lowenstein rules, Zimmermann identities
- 4. Short Distance Expansion

4.1. Principal part of the short distance expansion; 4.2. Example: The short distance expansion for  $\phi(x + y)\phi(x - y)$ ; 4.3. Asymptotic form of the short distance expansion; 4.4. Concluding Remarks

Appendix: The proof of Theorem 2

## 1. Introduction

Wilson's hypothesis [1] of the short distance expansion of products of composite operators plays a rather important role in a variety of contexts in field theory.

<sup>\*</sup> Supported by the Swiss National Science Foundation

Now at: Department of Mathematics, University of Virginia, Charloltesoille, VA 22903, USA