Commun. Math. Phys. 151, 67–87 (1993)

Communications in
Mathematical
Physics

Regularity of Weak Solutions to a Two-Dimensional Modified Dirac–Klein–Gordon System of Equations

Yuxi Zheng*

Courant Institute, New York University, New York, NY 10012, USA

Received March 6, 1992

Abstract. We show that solutions to the modified Dirac–Klein–Gordon system in standard notation

$$\begin{cases} (-i\gamma^\mu \partial_\mu + M)\psi = 0 \\ (-\Box + m^2)\varphi = g(t)\psi^\dagger \gamma^0 \psi \end{cases}$$

in two space dimensions with complex-valued initial data $\psi(0,x) \in L^2(\mathbb{R}^2; \mathbb{C}^4)$, real valued $\varphi(0,x) \in W^{1,2}(\mathbb{R}^2)$ and $\varphi_t(0,x) \in L^2(\mathbb{R}^2)$ have regularity

$$\psi^\dagger \gamma^0 \psi \equiv |\psi_1|^2 + |\psi_2|^2 - |\psi_3|^2 - |\psi_4|^2 \in \mathscr{H}^1_{\mathrm{loc}}(\mathbb{R}^3),$$
$$\varphi \in L^\infty_{\mathrm{loc}}(\mathbb{R}^1_+; L^2(\mathbb{R}^2)).$$

Here $\mathscr{H}^1_{\mathrm{loc}}(\mathbb{R}^3)$ denotes the (local) Hardy space, and $g(t)$ is assumed to be in $C^1(\mathbb{R})$ and $g(0) = 0$. Consequently nonlinear terms $\varphi\psi$ which appear in the classical coupled Dirac–Klein–Gordon system (with the modification $g = g(t) \in C^1$ and $g(0) = 0$) can then be defined in $L^\infty_{\mathrm{loc}}(\mathbb{R}^1_+; L^1(\mathbb{R}^2))$. We hope these results will be useful in establishing the existence of weak solutions to the classical coupled Dirac–Klein–Gordon system in the framework of compensated compactness.

Table of Contents

* Research at MSRI and IAS supported in part by NSF Grant DMS-8505550 and DMS-9100383