Commun. Math. Phys. 151, 53–66 (1993)

Communications in Mathematical Physics

On the Topology of Black Holes*

Gregory J. Galloway
Department of Mathematics and Computer Science, University of Miami, Coral Gables, FL 33124, USA

Received March 19, 1992

Dedicated to Ted Frankel

Abstract. We establish from local hypotheses some results concerning the final state topology of black holes. We show that the surface of a black hole must have 2-sphere topology and that the topology of space in its vicinity is simple.

1. Introduction

The classical theory of black holes (as elucidated, for example, in [HE]) imposes at the outset numerous global asymptotic conditions. For instance, spacetime is required to be weakly asymptotically simple and empty (which implies the existence of past and future null infinity $\mathscr{I}^{\pm}$) and future asymptotically predictable from a partial Cauchy surface S (which asserts that cosmic censorship holds). Certain global topological assumptions are imposed, as well (cf. [HE], p. 317). Although the purpose of these conditions is to model the region of spacetime in the vicinity of an isolated, or quasi-isolated, gravitating system, mathematically, they are conditions on all of spacetime. Since the formation of a black hole due to the gravitational collapse of some stellar object is viewed as a rather local phenomenon, it would seem to be of interest to consider what properties of black holes can be derived from purely local hypotheses (i.e., from assumptions made on an arbitrarily small region of spacetime in the vicinity of the black hole). The aim of this paper is to establish from local hypotheses some results concerning the final state topology of black holes. We show that, in the steady state limit, the surface of a black hole must have 2-sphere topology (thus recovering Hawking's result ([HE], p. 325) in the static case from local hypotheses) and that the region of space in the vicinity of the black hole is correspondingly simple. Before giving a precise statement of these results we briefly describe the philosophy behind them. Numerous examples and theorems (e.g. [G, L, FG, MA]) indicate that nontrivial spatial topology cannot support a state of gravitational equilibrium, and, in fact, tends to

* This work has been partially supported by NSF grants DMS-8802877 and DMS-9006678