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The Classification of Differential Structures on Quantum 2-Spheres

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Abstract. Exterior algebras of differential forms on quantum 2-spheres S^2_{qc}, $q \in [-1, 1] \backslash \{0\}$, $c \in [0, \infty]$ ($c = 0$ for $q = \pm 1$), are classified. In the definition of exterior algebras we assume the invariance w.r.t. the action of the quantum $SU(2)$ group and "dimensionality conditions" (which imply that we deal with "two-dimensional manifolds"). The exterior algebras exist only for $c = 0$ and are unique in that case. The corresponding generalized directional derivatives are provided.

0. Introduction

One of the most important problems of theoretical physics is to find a consistent theory which would generalize both the general theory of relativity and quantum field theory. In the opinion of some physicists, in such a future theory functions on space-time should be replaced by operators belonging to a non-commutative algebra. In other words, space-time should be replaced by a quantum space. (The basic idea could be that the laws of physics should be the same in each quantum space-time.) Therefore, it is important to investigate the properties of quantum spaces, especially those properties which could be important for physics, like the existence of differential structures. In addition to general considerations (cf. e.g. [W 1, C, W 3, W 5, Mau]) we need also concrete examples (cf. e.g. [W 2, M, RTF, PW, CSSW, CSW, WZ]). One of them is given by the quantum spheres S^2_{qc} [P 1], which are homogeneous spaces of quantum $SU(2)$ groups $SU_q(2)$ [W 2]. Quantum spheres are generalizations of the standard 2-sphere S^2 endowed with a classical right action of $SU(2)$ [or $SO(3)$]. (This action plays an important role in the description of spherical symmetric, stationary systems in physics, such as the hydrogen atom in quantum mechanics or the Schwarzschild solution in the general theory of relativity.)

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