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Quantum Knizhnik-Zamolodchikov Equations and Affine Root Systems

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Abstract. Quantum (difference) Knizhnik-Zamolodchikov equations [S1, FR] are generalized for the R-matrices from [Ch1] with the arguments in arbitrary root systems (and their formal counterparts). In particular, QKZ equations with certain boundary conditions are introducted. The self-consistency of the equations from [FR] and the cross-derivative integrability conditions for the r-matrix KZ equations from [Ch2] are obtained as corollaries. A difference counterpart of the quantum many-body problem connected with Macdonald's operators is defined as an application.

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0. Introduction

In a recent paper [FR], the so-called quantum R-matrices (solutions of the Yang-Baxter equations) were used to introduce certain systems of difference equations. Their quasi-classical limits are the r-matrix Knizhnik-Zamolodchikov equations defined in [Ch2] (see also [Ch3]). To be more precise, the systems of differential equations from the latter are connected with the root systems $(A, B, \ldots, G)$ describing the structure of the arguments. The construction from [FR] corresponds to the r-matrix equations with

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