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Quasi-Quantum Groups, Knots, Three-Manifolds, and Topological Field Theory

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Abstract. We show how to construct, starting from a quasi-Hopf algebra, or quasi-quantum group, invariants of knots and links. In some cases, these invariants give rise to invariants of the three-manifolds obtained by surgery along these links. This happens for a finite-dimensional quasi-quantum group, whose definition involves a finite group G, and a 3-cocycle ω, which was first studied by Dijkgraaf, Pasquier, and Roche. We treat this example in more detail, and argue that in this case the invariants agree with the partition function of the topological field theory of Dijkgraaf and Witten depending on the same data G, ω.

1. Introduction

It is by now well established that there are deep connections between two-dimensional rational conformal field theories (RCFT), three-dimensional topological field theories (TFT), and quantum groups when q is a root of unity, see e.g. [1–9].

A key element in any attempt at understanding these connections is the fact that both RCFT and quantum groups are sources of topological invariants of knots, links, and three-dimensional manifolds (through the TFT reinterpretation of RCFT). For instance, the invariants of the Hopf link are the elements of the matrix S [1, 2], and consideration of a chain of three circles is the key to proving Verlinde's formula. The construction of invariants of links from the representation theory of quantum groups was developed in [10–12]. In its most general form it appears in [12], where the concept of ribbon Hopf algebras is introduced. Examples of ribbon Hopf algebras are the "usual" quantum groups [13] $U_q\mathscr{G}$, where $\mathscr{G}$ is a semi-simple Lie algebra [7], the double $D(G)$ of a finite group G, and many more are discussed in a recent paper of Kauffman and Radford [14]. To our taste, the above connections are best explained in [12], where a TFT, formalized in

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