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Deformation Estimates for the Berezin-Toeplitz Quantization

L. A. Coburn★

Department of Mathematics, State University of New York at Buffalo, Buffalo, NY 14214, USA

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Abstract. Deformation estimates for the Berezin-Toeplitz quantization of $\mathbb{C}^n$ are obtained. These estimates justify the description of CCR $+\mathscr{K}$ as a first-order quantum deformation of AP $+C_0$, where CCR is the usual C^*-algebra of (boson) canonical commutation relations, $\mathscr{K}$ is the full algebra of compact operators, AP is the algebra of almost-periodic functions and C_0 is the algebra of continuous functions which vanish at infinity.

1. Introduction

We consider the family of Gaussian probability measures

$$d\mu_r(z) = \left(\frac{r}{\pi}\right)^n e^{-r|z|^2} dv(z)\,, \qquad r > 0$$

for $z = (z_1, \ldots, z_n)$ in complex Euclidean space $\mathbb{C}^n$, $dv(z)$ ordinary Lebesgue measure, $|z|^2 = |z_1|^2 + \ldots + |z_n|^2$. The space of entire $d\mu_r$-square-integrable functions is denoted by $H^2(d\mu_r) \equiv H^2(\mathbb{C}^n, d\mu_r)$. For g in $L^2(d\mu_r)$, the Berezin-Toeplitz operator $T_g^{(r)}$ is defined on a dense linear subspace of $H^2(d\mu_r)$ by

$$(T_g^{(r)}h)(z) = \int g(w)h(w)e^{rz\cdot w} d\mu_r(w)\,.$$

Here $z \cdot w \equiv z_1\bar{w}_1 + \ldots + z_n\bar{w}_n$ and $e^{rz\cdot w}$ is the Bergman reproducing kernel for $H^2(d\mu_r)$ so that, for gh in $L^2(d\mu_r)$, $T_g^{(r)}h$ is in $H^2(d\mu_r)$.

The map $g \to T_g^{(r)}$ has been considered by Berezin [Be] and others [Ba, G, Ho] as a "quantization" in which r plays the role of the reciprocal of Planck's constant.

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