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Limit Behavior of Saturated Approximations of Nonlinear Schrödinger Equation

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Abstract. We consider the solution $u_\varepsilon(t)$ of the saturated nonlinear Schrödinger equation

$$i\,\partial u/\partial t = -\Delta u - |u|^{4/N}u + \varepsilon|u|^{q-1}u \quad \text{and} \quad u(0,.) = \varphi(.)\,, \tag*{$(1)_\varepsilon$}$$

where $N \geq 2$, $\varepsilon > 0$, $1+4/N < q < (N+2)/(N-2)$, $u : \mathbb{R}\times\mathbb{R}^N \to \mathbb{C}$, φ is a radially symmetric function in $H^1(\mathbb{R}^N)$. We assume that the solution of the limit equation is not globally defined in time. There is a $T > 0$ such that $\lim\limits_{t\to T} \|u(t)\|_{H^1} = +\infty$, where $u(t)$ is the solution of

$$i\,\partial u/\partial t = -\Delta u - |u|^{4/N}u \quad \text{and} \quad u(0,.) = \varphi(.)\,. \tag{1}$$

For $\varepsilon > 0$ fixed, $u_\varepsilon(t)$ is defined for all time. We are interested in the limit behavior as $\varepsilon \to 0$ of $u_\varepsilon(t)$ for $t \geq T$. In the case where there is no loss of mass in u_ε at infinity in a sense to be made precise, we describe the behavior of u_ε as ε goes to zero and we derive an existence result for a solution of (1) after the blow-up time T in a certain sense. Nonlinear Schrödinger equation with supercritical exponents are also considered.

I. Introduction

In the present paper, we consider the saturated nonlinear Schrödinger equation:

$$i\partial u/\partial t = -\Delta u - |u|^{4/N}u + \varepsilon|u|^{q-1}u \quad \text{and} \quad u(0,.) = \varphi(.)\,, \tag*{$(1)_\varepsilon$}$$

where Δ is the Laplace operator on $\mathbb{R}^N$, $u : [0,T) \times \mathbb{R}^N \to \mathbb{C}$, and $\varphi \in H^1(\mathbb{R}^N)$. We assume that $N \geq 2$, $\varepsilon > 0$ and $1 + 4/N < q < (N+2)/(N-2)$.