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Dressing Symmetries

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Abstract. We study the group of dressing transformations in soliton theories. We show that it is generated by the monodromy matrix. This provides a new proof of their Lie–Poisson property. We treat in detail the examples of the Toda field theories and the Heisenberg model. We show that the group of dressing transformations is the classical precursor of the various manifestations of quantum groups in these models, e.g. algebraic Bethe ansatz, non-local currents, or quantum group symmetries. Finally, we define field multiplets supporting a linear representation of the dressing group and we show that their exchange algebras are encoded in the classical double.

1. Introduction

Are quantum integrable models reducible to quantum group theory? This assertion is best examplified by the algebraic formulation of two-dimensional conformal field theories [1] but remains uncertain for massive integrable models. On one hand, quantum groups [2, 3] and their representation theory are now well understood although some of their relations with the quantum algebraic Bethe Ansatz are still mysterious. On the other hand, a large number of methods for studying integrable models have been developed, cf. e.g. [4, 5], including the most famous algebraic quantum inverse scattering method [6]. More recently, new developments have suggested the use of non-local symmetries for reformulating integrable models as quantum group theories [7, 8, 9]. However, despite suggestive facts, the assertion is still not established by any of these approaches.

The aim of this paper is to try to understand the occurrence of non-local symmetries in classical soliton equations and their relations with semi-classical analogues of quantum group symmetries. We will deal with integrable soliton

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