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A Family of Metrics on the Moduli Space of $\mathbf{CP}^2$ Instantons

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Abstract. A family of Riemannian metrics on the moduli space of irreducible self-dual connections of instanton number $k = 1$ over $\mathbf{CP}^2$ is considered. We find explicit formulas for these metrics and deduce conclusions concerning the geometry of the instanton space.

1. Introduction

Let $\mathscr{N}^+$ be the space of gauge equivalence classes of irreducible self-dual connections on a principal $SU(2)$-bundle P over a Riemannian 4-manifold M. Define a Riemannian metric g^s on $\mathscr{N}^+$ for $s \geq 0$ by

$$(g^s)_{[Z]}(u_1, u_2) = ((1 + s\Delta_Z)u_1\, (1 + s\Delta_Z)u_2)\,,$$

where $[Z] \in \mathscr{N}^+$ and $(\,,)$ denotes the L^2-product. Then g^0 is the usual L^2-metric, whereas g^s is induced by a strong Riemannian metric on the orbit space of all irreducible connections on P for $s > 0$.

Results concerning the L^2-metric g^0 when M is the standard 4-sphere S^4 and the instanton number $k(P)$ is 1 were obtained by several authors (see [5, 8, 10]). In particular, it was shown that

(i) $(\mathscr{N}^+, g^0)$ is incomplete and has finite diameter and volume.
(ii) The completion of $(\mathscr{N}^+, g^0)$ differs from $\mathscr{N}^+$ by a set diffeomorphic to S^4.

Groisser and Parker generalized these results and established some other general properties of g^0 under certain topological assumptions on M and P (cf. [9]).

In [2] we examined the family $\{g^s\}_{s\geq 0}$ in the S^4 example. We showed that $(\mathscr{N}^+, g^s)$ is complete and has infinite diameter and volume for $s > 0$.

In the present paper we will be concerned with the case that M is $\mathbf{CP}^2$ and $k(P) = 1$. Then the moduli space $\mathscr{N}$ of self-dual connections is topologically a cone