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Statistics of Shocks in Solutions of Inviscid Burgers Equation

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Abstract. The purpose of this paper is to analyze statistical properties of discontinuities of solutions of the inviscid Burgers equation having a typical realization $b(y)$ of the Brownian motion as an initial datum. This case was proposed and studied numerically in the companion paper by She, Aurell and Frisch. The description of the statistics is given in terms of the behavior of the convex hull of the random process $w(y) = \int_0^y (b(\eta) + \eta)\, d\eta$. The Hausdorff dimension of the closed set of those y where the convex hull coincides with w is also studied.

1. General Properties of Solutions of the One-dimensional Inviscid Burgers Equation

Burgers equation is one of the most popular non-linear equations which appears in many concrete physical problems. In this paper we study some properties of solutions of the inviscid Burgers equation having as initial velocity a typical realization of the Brownian motion (as a function of the space variable). This case was proposed in a companion paper by She, Aurell, and Frisch [1] where one can find physical motivations for this case as well as many qualitative arguments and numerical results.

We start with the geometric description of the process of construction of solutions to the inviscid Burgers equation. This theory was already exposed in the pioneering works of Hopf (see [2]) and Burgers (see [3]). We present here a slightly different approach compared with [2] and [3]. The companion paper [1] also begins with this analysis. The notations in the present paper and in [1] are slightly different but it is easy to establish the correspondence between them.

We recall that the one-dimensional Burgers equation without force has the form

$$\partial_t u + u\partial_x u = \mu \partial_x^2 u, \qquad -\infty < x < \infty\,.$$