Commun. Math. Phys. 148, 425–444 (1992)

Communications in
Mathematical
Physics

The Asymptotic Behavior of Yang–Mills Fields in the Large

Vladimir Georgiev[1] **and Pedro Paulo Schirmer**[2]*

[1] Institute of Mathematics, Bulgarian Academy of Sciences, Boul. Acad. Bonchev bl.8, Sofia 1113, Bulgaria

[2] Institut für Angewandte Mathematik der Universität Bonn, Wegelerstrasse 10, W-5300 Bonn 1, Federal Republic of Germany

Received May 14, 1990; in revised form August 2, 1991

Abstract. We consider Yang–Mills fields in Minkowski space-time and prove that all spherically symmetric solutions in the canonical gauge decay in time, provided the initial data has finite conformal energy.

1. Introduction

We consider Yang–Mills equations in Minkowski Space-Time R^{3+1}:

$$F_{\alpha\beta}{}^{;\beta} = 0\,, \tag{1.1}$$

$$^{*}F_{\alpha\beta}{}^{;\beta} = 0\,. \tag{1.2}$$

F_A: $R^{3+1} \to \Lambda^2\mathscr{G}$ is the Yang–Mills curvature tensor of a Yang–Mills potential A: $R^{3+1} \to \Lambda^1\mathscr{G}$ and $\mathscr{G}$ is the Lie algebra of the gauge group G. System 1.1–1.2 is a non-linear hyperbolic system of partial differential equations after the choice of a gauge. The question of global existence has been settled already and one obtains global solutions in H^s (cf. [4]) but no information about the asymptotic behavior of its solutions was obtained. It was proved later in [1] (see also [2]) the existence of global large solutions in the weighted Sobolev spaces $H^{s,\delta}$ together with the characterization of the asymptotic behavior in time. The major drawback though is the strong fall-off rate of the Cauchy data, requiring for example that the electric field decays like $E(0,x) = O(|x|^{-4})$ as $|x| \to +\infty$. This excludes configurations containing Coulomb charges and also dipole-type waves. In the special case of small-amplitude solutions one can obtain estimates (cf. [3, 11, 12 and 13]) which give the long-term behavior in time of the solutions.

The purpose of this work is to investigate solutions corresponding to dipole-type Cauchy data and to provide the time-asymptotics in the large amplitude sector. Our results apply to generic spherically symmetric Yang–Mills fields in the so-called canonical gauge ([12]):

$$A_0 = \sum_{l=1}^{N} \phi_l(t,r)\wp_l\,, \tag{1.3}$$

* Work supported by Sonderforschungsbereich 256 of the Deutsche Forschungsgemeinschaft