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On the Integrability of the Super-KdV Equation

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Abstract. Supersymmetric analogues of the Gelfand–Dikii polynomials are derived. An alternative proof of the super-KdV equation is given using identities obeyed by the polynomials. They also allow the recursive generation of the infinite number of conserved quantities for the super-KdV flow.

1. Introduction

The Korteweg–de Vries (KdV) equation is a nonlinear differential equation describing the "time" evolution of a function $u(x)$ of one "spatial" variable,

$$\partial_t u = \frac{1}{2}(\partial^3 u - 6u\,\partial u)\,, \tag{1.1}$$

where $\partial = \partial_x$. This equation is integrable, in that there exist infinitely many conserved quantities, and it is intimately related to the differential operator $L = -\partial^2 + u$. Recently, the KdV hierarchy has been found to be related to matrix models [1] and to two-dimensional topological gravity [2]. Central to this interpretation are the Gelfand–Dikii polynomials [3] and the recursion relations they obey. The string equations of matrix models and the correlation functions of topological gravity are conveniently expressed in terms of these polynomials.

In the supersymmetric version of the KdV equation (the sKdV equation), the variable x acquires a Grassmann partner θ, so $X \equiv (x, \theta)$ are coordinates in a one dimensional superspace. The sKdV equation is a nonlinear differential equation describing the "time" evolution of a Grassmann-valued superfield $\hat{U}(X) = \hat{u}(x) + \theta u(x)$, where $u(x)$ is an ordinary function and $\hat{u}(x)$ is a Grassmann-valued function (hats denote Grassmann-valued quantities). Defining the operator $D = \partial_\theta + \theta\partial$, the sKdV equation is [4, 5, 6]:

$$\partial_t \hat{U} = \frac{1}{2}(\partial^3 \hat{U} - 3(\partial\hat{U})D\hat{U} - 3\hat{U}\partial D\hat{U})\,. \tag{1.2}$$

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