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# Large Deviations for Multiplicative Chaos

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**Abstract.** The local singularities for a class of random measures, obtained by random iterated multiplications, are investigated using the thermodynamic formalism. This analysis can be interpreted as a rigorous study of the phase transition of a system with random interactions.

## 1. Introduction

The notion of singularity of a measure was introduced in [6] in connection with potential theory. A typical singularity can be defined when it is associated to the capacity of the measure. It was recently realized that this analysis can be further developed for the invariant measure of some dynamical systems [2]. In this special situation, one can use expansive (respectively contractive) properties of the mapping to compare the singularities of a measure at different scales. Using the thermodynamic formalism, it was possible to obtain non-trivial results on the local singularities of a measure. This information is usually recorded in the so-called $f(\alpha)$ functions which can be defined as follows. Let $\mu$ be a Borel measure on $\mathbf{R}^d$. For any point $M$ in $\mathbf{R}^d$ we define two numbers $\alpha_+$ and $\alpha_-$ by

$$\alpha_+(M) = \limsup_{\varepsilon \to 0} \frac{\log|\mu(B_\varepsilon(M))|}{d \log \varepsilon} \quad \text{and} \quad \alpha_-(M) = \liminf_{\varepsilon \to 0} \frac{\log|\mu(B_\varepsilon(M))|}{d \log \varepsilon},$$

where $B_\varepsilon(M)$ is the ball centered at $M$ of radius $\varepsilon$. The numbers $\alpha_+(M)$ and $\alpha_-(M)$ describe the local singularity of the measure $\mu$ at $M$. For simple ergodic dynamical systems the two functions $\alpha_+$ and $\alpha_-$ are equal and constant $\mu$-almost surely (see [12] for a more general situation). It was however realised in [8] that there is some interesting information in the sets where $\alpha_+$ and $\alpha_-$ take a value $\alpha$ different from the typical one. If one defines the level sets $B_\alpha^\pm$ by

$$B_\alpha^\pm = \{M \mid \alpha_\pm(M) = \alpha\},$$

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