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# Semirigid Geometry

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**Abstract.** We provide an intrinsic description of N-super Riemann surfaces and TN-semirigid surfaces. Semirigid surfaces occur naturally in the description of topological gravity as well as topological supergravity. We show that such surfaces are obtained by an integrable reduction of the structure group of a complex supermanifold. We also discuss the supermoduli spaces of TN-semirigid surfaces and their relation to the moduli spaces of N-super Riemann surfaces.

## 1. Introduction

Semirigid surfaces have been shown [1, 2] to provide a geometric framework to describe $2d$ topological gravity and supergravity. For example, in the simplest theory the dilaton as well as the puncture equations have been proven using the semirigid formalism [3, 4]. In this paper, we provide an intrinsic or coordinate invariant definition of semirigid super Riemann surfaces (SSRS) as well as ordinary super Riemann surfaces (SRS). The discussion of SRS is a natural extension to similar discussions provided in [5] and applied in [6] for the case of $N = 1$ SRS and in [7] for $N = 2$; the framework follows Cartan's theory of $G$-structures. (For an introduction to $G$-structures, see for example [8, 9, 6, 10].) We show that these structures subject to some conditions called "torsion constraints" are integrable, which relates our intrinsic definition to the coordinate dependent definitions.

We will first discuss the various definitions and illustrate $G$-structures via two examples in Sect. 2. We also find the appropriate group $G$ for superconformal and semirigid surfaces and the corresponding torsion constraints. Section 3 deals with showing that the $G$-structures we impose are integrable provided the constraints are satisfied. Briefly the results are as follows. If we begin with a complex supermanifold, then N-SRS have no essential torsion constraints, generalizing Baranov, Frolov, and Schwarz [11], who considered $N = 1$.[1] We will refer to semirigid surfaces with N-supersymmetry as "topological N-SRS," or TN for short. TN $= 0$ surfaces have a rather trivial essential constraint while TN $= 1$ surfaces have

[1] This generalization was asserted in the appendix to [12]. The constraints found in [13] and discussed in [6] arise when we begin with a *real* supermanifold