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A Global Formalism for Nonlinear Waves in Conservation Laws*

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Abstract. We introduce a unifying framework for treating all of the fundamental waves occurring in general systems of n conservation laws. Fundamental waves are represented as pairs of states statisfying the Rankine-Hugoniot conditions; after trivial solutions have been eliminated by means of a blow-up procedure, these pairs form an $(n+1)$-dimensional manifold $\mathscr{W}$, the fundamental wave manifold. There is a distinguished n-dimensional submanifold of $\mathscr{W}$ containing a single one-dimensonal foliation that represents the rarefaction curves for all families. Similarly, there is a foliation of $\mathscr{W}$ itself that represent shock curves. We identify other n-dimensional submanifolds of $\mathscr{W}$ that are naturally interpreted as boundaries of regions of admissible shock waves. These submanifolds also have one-dimensional foliations, which represent curves of composite waves. This geometric framework promises to simplify greatly the study of the stability and bifurcation properties

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