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# Finite-Volume Statistical Mechanics of Two-Component Coulomb-like Systems and the Principle of Macroscopic Equivalence

**Michael K.-H. Kiessling**

Courant Institute of Mathematical Sciences, New York University, New York, NY 10012, USA

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**Abstract.** Classical stable charge-symmetric two-component systems are discussed in a fixed domain $\Lambda \subset \mathbb{R}^d$. The limit $N \to \infty$ of the finite-$N$ canonical Gibbs ensemble is compared with the results obtained from a discussion of the Gibbs measures on the space of infinite configurations (the states). A first-order phase transition in the Gibbs states is proved for a large class of interactions, including regularized Coulomb interactions for $d \geqq 3$. In the latter case the transition is isomorphic to an implosion/explosion transition in regularized gravitational systems. Spherical symmetry is not assumed. A transition occurs for certain large-domain/low-temperature pairs $(\Lambda, \beta^{-1})$, but ceases to exist in the infinite-volume ensemble. The phase transition supports the conjecture that the standard thermodynamic-limit sequence can be nonuniform even for standard $H$-stable Hamiltonians. The results about the limit of the finite-$N$ ensemble are less complete due to lack of sufficient control of the correlations. However, some notable differences between both descriptions are shown, which are caused by noncommuting limits. Possible physical consequences and open questions are pointed out.

## 1. Introduction

For systems with suitably restricted interactions, e.g., stability and temperedness may be required, equilibrium thermodynamics is normally *defined* in terms of the infinite volume thermodynamic limit. The monographs by Ruelle [Rue 1] and Balescu [Bal], for instance, give an overview over the general ideas and rigorous results. After many remarkable achievements about the existence of the T-limit, e.g., [vHo, Yan-Lee, Lee-Yan, Rue 2, 3, Fis, Gri 1, 2, Lie-Leb, Frö-Par 1, 2], one reasonable next step is to inquire into the fundamental problem of how big a system must be in order to belong to the asymptotic regime of the T-limit. According to the physics folklore, $10^{23} \approx \infty$, emphasizing the laboratory evidence. It seems to be a challenging task to give rigorous estimates for the