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Structures of S-Matrices for Three Body Schrödinger Operators

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Abstract. Structures of the S-matrix associated with the collision process from 2 clusters to 3 clusters are studied. This S-matrix is shown to have a continuous kernel except for 2-dimensional spheres on which 2-body subsystems have zero velocity. On these spheres, the S-matrix has, in general, singularities whose existence arises from the zero eigenvalues and the zero resonances of the 2-body subsystems.

1. Introduction

1.1 Collision Process in the Three-Body Problem. We consider collision processes of quantum mechanical three particles labelled by 1, 2, 3. Suppose in the initial state the two of them form a bound state, denoted by (1, 2), and the third particle collides with this pair. Then it follows from the asymptotic completeness of the wave operators (see e.g. Enss [4], Sigal-Soffer [20] or Graf [7]) that there occurs one of the following five phenomena:

$$(1,2)+(3) \Rightarrow \begin{cases} \text{(a)} & (1,2)+(3), \\ \text{(b)} & (1,2)^*+(3)_*, \\ \text{(c)} & (1,2)'+(3), \\ \text{(d)} & (1,3)+(2), \\ \text{(e)} & (1)+(2)+(3). \end{cases}$$

(a) is an elastic process. In (b), the energy of the pair changes. In (c), the energy of the pair does not change, but this pair takes a different state (which happens when the eigenvalue is degenerate). (d) is a rearrangement process. Finally in (e), all of the three particles move freely after the collision. The first four cases are treated in essentially the same way as in the 2-body problem. In this paper, we study properties of the S-matrix associated with the case (e).

1.2 S-matrix. In $\mathbf{R}^3$ we consider three particles with mass m_i and position x^i. We