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Inertial Range Scaling of Laminar Shear Flow as a Model of Turbulent Transport

Qiang Zhang* and James Glimm**

Department of Applied Mathematics and Statistics, SUNY at Stony Brook, Stony Brook, NY 11794-3600, USA

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Abstract. Asymptotic scaling behavior, characteristic of the inertial range, is obtained for a fractal stochastic system proposed as a model for turbulent transport.

1. Introduction

The determination of the statistical behavior of a fluid from the statistical properties of a random velocity field is important in the study of tracer flow in heterogeneous porous media, ground water ecology, and fully developed turbulence. In earlier work [4, 9], the asymptotic scaling exponents were obtained for the motion of a fluid determined by a convection-diffusion equation

$$T_t + \vec{v}(\vec{x}, t) \cdot \nabla T = \mu \Delta T, \quad T(0, \vec{x}) = T_0(\vec{x}), \tag{1.1}$$

with $\mu = 0$. Here T is a physical quantity, $\vec{v}$ is a random velocity field, and μ is the molecular diffusion coefficient. The purpose of this paper is to relate these exponents to the similar but in some cases distinct exponents of [1], obtained for the equation

$$T_t + v(x, t) T_y = \mu \Delta T, \quad T(0, x, y) = T_0(x, y). \tag{1.2}$$

Our results are either independent of cutoffs, (i.e. infrared finite) and thus are properties of the self similar, or inertial, regime, or else the cutoff cannot be removed, because the inertial range scaling behavior is infrared divergent. In the infrared divergent case, the cutoff dependence of the scaling exponents is exhibited explicitly.

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