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$\bar{\partial}$-Torsion, Foliations and Holomorphic Vector Fields

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Abstract. In this paper, we investigate the relation between $\bar{\partial}$-torsion and holomorphic vector fields. We consider complex manifolds which fibrate over the torus having a transverse one dimensional holomorphic foliation. The torsion of the total space is then computed in terms of compact leaves. This can be interpreted as a Lefschetz formula for flows of holomorphic vector fields.

1. Introduction

In analogy of recent work of Fried [Fr 3–5], Ruelle [Ru] and older work of Ray and Singer [RS], we will relate topological invariants of complex manifolds to the closed orbits of vector fields or closed leaves of a foliation. We will consider fibrations of compact complex manifolds M

$$\begin{array}{ccc} F & \longrightarrow & M \\ & & \big\downarrow{\scriptstyle \pi} \\ & & B, \end{array} \tag{1.1}$$

where $B=\mathbb{C}/\Gamma$, $\Gamma=\mathbb{Z}+\tau\mathbb{Z}$ with $\operatorname{Im}\tau>0$ together with a complex one dimensional holomorphic foliation $\mathscr{F}$ transverse to the fibration. When M is Kähler, Crew and Fried [CF] showed that the existence of a non-vanishing holomorphic vector field on M gives rise to such a foliated fibration. The leaves of $\mathscr{F}$ are then spanned by the flow of the vector field. It should be stressed that in this case, the projection map is not necessarily holomorphic. We will restrict ourselves to the case where π is holomorphically locally trivial.

The invariants one consider are ratios of $\bar{\partial}$-torsion (see [RS]). Given two flat acyclic bundles $\mathscr{E}_1$ and $\mathscr{E}_2$ over B of the same rank, one can construct natural metric invariants of M, namely the ratios

$$\frac{T_p(M,\pi^*\mathscr{E}_1)}{T_p(M,\pi^*\mathscr{E}_2)}, \qquad p=0,\ldots,\dim_{\mathbb{C}}M, \tag{1.2}$$