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Finite-Size Scaling and Surface Tension from Effective One Dimensional Systems

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Abstract. We develop a method for precise asymptotic analysis of partition functions near first-order phase transitions. Working in a $(\nu+1)$-dimensional cylinder of volume $L \times \cdots \times L \times t$, we show that leading exponentials in t can be determined from a simple matrix calculation provided $t \geqq v \log L$. Through a careful surface analysis we relate the off-diagonal matrix elements of this matrix to the surface tension and L, while the diagonal matrix elements of this matrix are related to the metastable free energies of the model. For the off-diagonal matrix elements, which are related to the crossover length from hypercubic ($L = t$) to cylindrical ($t = \infty$) scaling, this includes a determination of the pre-exponential power of L as a function of dimension. The results are applied to supersymmetric field theory and, in a forthcoming paper, to the finite-size scaling of the magnetization and inner energy at field and temperature driven first-order transitions in the crossover region from hypercubic to cylindrical scaling.

1. Introduction

Finite-size effects at first-order phase transitions exhibit a rich set of phenomena that have been brought to light by a number of authors [FB, P, PF]. Recent rigorous studies have clarified some of the issues involved with the rounding of the magnetization jump in a finite periodic hypercube with periodic boundary conditions [BK, BKM]. One may analyze the asymptotics of the partition function or the magnetization as the dimension, $\mathscr{L}$, of the hypercube tends to infinity. For example, varying the magnetic field μ of the Ising model in a neighborhood of

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