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Vortex Scattering

T. M. Samols

Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Silver Street, Cambridge CB3 9EW, and King's College, Cambridge, CB2 1ST, United Kingdom

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Abstract. The geodesic approximation to vortex dynamics in the critically coupled abelian Higgs model is studied. The metric on vortex moduli space is shown to be Kähler and a scheme for its numerical computation described. The scheme is applied to the 2-vortex system and the geodesic scattering compared with previous simulations of the full field theory. The quantum scattering is also analysed.

1. Introduction

Describing the dynamics of field theory solitons is in general a difficult problem. Classically, it requires that one solve the initial value problem for a set of non-linear hyperbolic partial differential equations. Although there are some very special (exactly-integrable) systems for which explicit time-dependent multisoliton solutions can be constructed – for instance, the sine-Gordon model – no such systems enjoying Lorentz-invariance have been found in more than one space dimension. In more physically interesting cases one must resort to numerical simulation or work within some kind of approximation scheme.

One possibility, at low energies, is that most of the degrees of freedom of the fields remain unexcited and the field theory can be well approximated by a finite-dimensional system. Truncating the field theory in this way is usual in the collective coordinate description of a single soliton. That it might be appropriate to the description of several strongly interacting solitons was first proposed by Manton [1] in connection with the scattering of Bogomol'nyi-Prasad-Sommerfield (BPS) monopoles. This theory is one of a class admitting static multisoliton solutions corresponding to arbitrary configurations of solitons at rest. The existence of the solutions may be understood physically as due to the absence of static forces between separated solitons. Mathematically, the essential property appears to be that they saturate a topological lower bound on the field energy and as a consequence satisfy a first order field equation (Bogomol'nyi equation) [2]. Manton's idea is that in such theories the low energy dynamics of several solitons may – just as for a single soliton – be approximated by motion on the space of the corresponding static solutions.