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Instantons and Mirror K3 Surfaces

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Abstract. The instanton moduli space of a real 4-dimensional torus is an 8-dimensional Calabi-Yau manifold. Associated to this Calabi-Yau manifold are two (singular) K3 surfaces, one a quotient, the other a submanifold of the moduli space; both carry a natural Calabi-Yau metric. They are curiously related in much the same way as special examples of complex 3-dimensional mirror manifolds; however, in our case the "mirror" is present in the form of instanton moduli.

1. Introduction

In the study of connections on a bundle $P \to \mathbb{T}$ over a Riemannian 4-manifold $\mathbb{T}$ the object of primary interest is the moduli space $\mathcal{M}(P)$ of anti-self-dual connections on P. These moduli spaces inherit various structures from $\mathbb{T}$: when $\mathbb{T}$ is a projective algebraic variety, $\mathcal{M}(P)$ is quasiprojective, and when $\mathbb{T}$ carries a hyperkähler metric then so does $\mathcal{M}(P)$.

An important class of 4-manifolds are the flat tori, and in [BMT] it was investigated what the structure of $\mathcal{M}(P)$ is when the bundle P on $\mathbb{T}$ satisfies

$$p_1(P) = -4, \quad w_2(P) \neq 0, \quad w_2(P)^2 = 0.$$

It was found that on $\mathcal{M}(P)$ we have a $\mathbb{T}$-action through translating connections and the quotient $\mathcal{M} = \mathcal{M}(P)/\mathbb{T}$ admits a compactification to a Todorov surface $\bar{\mathcal{M}} = \mathcal{M} \cup \infty$ with a natural hyperKähler metric induced from the L^2 metric on the space of connections on P. A Todorov surface is a K3 orbifold.

The moduli space $\mathcal{M}$ is not compact because instantons can bubble off, compare [FU], and the crux of the argument is that the hyperKähler metric extends over ∞, as an orbifold metric. The point ∞ is a D_4 singularity in $\bar{\mathcal{M}}$, and $\bar{\mathcal{M}}$ has another 12 singularities of type A_2. The latter arise as quotient singularities for the $\mathbb{T}$-action on $\mathcal{M}(P)$.

In this paper we shall study a further Todorov surface associated to $\mathcal{M}(P)$. First of all, $\mathcal{M}(P)$ can be compactified to a complex orbifold by adding a torus $\mathbb{T}_\infty$