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Reduction Formulae for Euclidean Lattice Theories

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Abstract. LSZ reduction formulae for Euclidean Lattice Theories are presented.

1. Introduction

There is an increasing interest in the physics literature in the study of scattering processes in Euclidean Lattice Field Theories (see f.i. [4, 5]). The existence of multi-particle states and of a well defined S-matrix for these theories has been established in [1] (see also [10]). An important question which rises in this context is the question of the existence of LSZ reduction formulae for the computation of S-matrix elements. This work is devoted to the rigorous derivation of these formulae. For relativistic quantum field theories they were first obtained by Lehmann, Symanzik, and Zimmermann in [3] under special assumptions concerning weak asymptotic limits of the fields. The first rigorous proof in the context of Wightman Quantum Field Theories was given by Hepp in [2]. Here we will follow essentially Hepp's ideas but some adaptations to our case are necessary due to the following two facts. First, [2] makes use of locality (Einstein causality) in a strong sense, a property which is generally not available for Lattice Theories. Second, the clustering of the Wightman functions obtained in [1] (see Theorem 2 below), which is an essential property for the construction of the scattering theory, is not uniform in the whole region of space-like separated points.

In Sect. 2 the notation used and the results of [1] which will be needed are introduced. Section 3 presents a smoothness theorem on suitably smeared expectations of time-ordered products of fields which leads directly to the reduction formulae in Sect. 4. The problems mentioned in the last paragraph manifest in the proof of this smoothness theorem, given in Subsects. 3.1 and 3.2. In this last subsection we show how to compensate the lack of locality with the use of the clustering property.

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