Commun. Math. Phys. 143, 501–525 (1992)

Communications in Mathematical Physics

A Special Class of Stationary Flows for Two-Dimensional Euler Equations: A Statistical Mechanics Description

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Received November 6, 1990; in revised form July 16, 1991

Abstract. We consider the canonical Gibbs measure associated to a N-vortex system in a bounded domain Λ, at inverse temperature $\tilde{\beta}$ and prove that, in the limit $N \to \infty$, $\tilde{\beta}/N \to \beta$, $\alpha N \to 1$, where $\beta \in (-8\pi, +\infty)$ (here α denotes the vorticity intensity of each vortex), the one particle distribution function $\varrho^N = \varrho^N(x)$, $x \in \Lambda$ converges to a superposition of solutions ϱ_β of the following Mean Field Equation:

$$\begin{cases} \varrho_\beta(x) = \dfrac{e^{-\beta\psi}}{\int\limits_\Lambda e^{-\beta\psi}}; \qquad -\Delta\psi = \varrho_\beta \text{ in } \Lambda \\ \psi|_{\partial\Lambda} = 0 . \end{cases} \tag{A.1}$$

Moreover, we study the variational principles associated to Eq. (A.1) and prove that, when $\beta \to -8\pi^+$, either $\varrho_\beta \to \delta_{x_0}$ (weakly in the sense of measures) where x_0 denotes an equilibrium point of a single point vortex in Λ, or ϱ_β converges to a smooth solution of (A.1) for $\beta = -8\pi$. Examples of both possibilities are given, although we are not able to solve the alternative for a given Λ. Finally, we discuss a possible connection of the present analysis with the 2-D turbulence.

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* Work partially supported by CNR (GNFM) and MPI