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Central Limit Theorems for the One-Dimensional Rayleigh Gas with Semipermeable Barriers

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Abstract. A version of the one-dimensional Rayleigh gas is considered: a point particle of mass M *(molecule)*, confined to the unit interval $[0, 1]$, is surrounded by an infinite ideal gas of point particles of mass 1 *(atoms)*. The molecule interacts with the atoms and with the walls via elastic collision. Central limit theorems are proved for a wide class of additive functionals of this system (e.g. the number of collisions with the walls and the total length of the molecular path).

1. Introduction

Since the discovery of Brownian motion, it is a great challenge to theoretical physicists and mathematicians to understand its dynamical theory. The chaotic behaviour of small particles in a liquid or in a gas seems to contradict the deterministic laws of the mechanics. Though the mathematical model (Wiener process) was constructed by Wiener and Lévy quite a time ago, the connection between the strict mathematical approach based on microscopic assumptions and the physical reality is far from being completely understood.

The Rayleigh gas, consisting of a heavy Brownian particle (molecule) interacting with an infinite ideal gas of light particles (atoms), seems to be a simple, nonetheless technically quite difficult real mechanical model of the Brownian motion. Here we summarize the known results on this and related models. The common feature of these models is that the randomness of the system is due to the ideal gas, while the possible recollisions give rise to a memory. The qualitative approach of the problem aimed at the ergodic properties of the Rayleigh gas. The simplest one-dimensional case was investigated in [5], where the molecule is confined to a unit interval. This restriction essentially reduced the memory of the system, and rendered possible the proof of the Bernoulli property.

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