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# Super Beltrami Differentials

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**Abstract.** Superconformal structures on a given 2|2-dim supermanifold give rise to the notion of super Riemann surfaces (SRS's). We investigate super Beltrami coefficients which parametrize (almost) superconformal structures on the supermanifold. The integrability condition of the structure reduces to a simple relation among the coefficients. Taking this into account, we can write the super Beltrami equations in a transparent form. Then an analysis of these equations enables us to prove the possibility of the special gauge choice of the Wess-Zumino type for the super Beltrami differentials. This gauge choice simplifies the description of the deformations of SRS's considerably and its existence will afford a better understanding of the structure of the super Teichmüller space.

## 0. Introduction

Two-dimensional field theories perceive the space-time which they inhabit to be a super Riemann surface (SRS) if their couplings to supergravity are superconformally invariant. The theory of SRS's plays an important role in the studies of the two-dimensional supergravity, superstring in the Neveu-Schwarz formalism and superconformal field theories. For example, holomorphy properties of the superstring measure on the moduli space of SRS's (super moduli space) are the fundamental importance of the theory [1].

There are mainly two ways in defining SRS's, that is, patch definition [2] (including algebraic definition [3]) and frame definition [4, 5], and for each of them there exist corresponding approaches to the moduli problem [3–6]. In this paper, we develop the study of the super Beltrami equations, clarifying the basic properties of the super Beltrami differentials and investigate the Teichmüller deformations of SRS's of genus $p>1$.

This approach is based on the frame definition of SRS's in which the notion of SRS is derived from 2-d supergravity geometry. In this respect, it is fit for the direct applications to physics [7–9]. The geometrical structure on 2|2-supermanifolds