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# On The Algebraic Structure of $N = 2$ String Theory

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**Abstract.** In $N = 2$ string theory the chiral algebra expresses the generations and anti-generations of the theory and the Yukawa couplings among them and is thus crucial to the phenomenological properties of the theory. Also the connection with complex geometry is largely through the algebras. These algebras are systematically investigated in this paper. A solution for the algebras is found in the context of rational conformal field theory based on Lie algebras. A statistical mechanics interpretation for the chiral algebra is given for a large family of theories and is used to derive a rich structure of equivalences among the theories (dihedralities). The Poincaré polynomials are shown to obey a resolution series which cast these in a form which is a sum of complete intersection Poincaré polynomials. It is suggested that the resolution series is the proper tool for studying all $N = 2$ string theories and, in particular, exposing their geometrical nature.

## 1. Introduction

In many respects, a viable physical theory is not unlike a basic mathematical one. Apart from being experimentally correct, all good physical theories are marked by a set of simple concepts and the depth and elegance of their results. The study of nature, from this viewpoint, may be considered as the uncovering of the principles from which it stems, with experimental data supplying the lead. String theory, although as yet only partially understood and thus hard to confront directly with the realm of particle physics and their interactions, passes the criteria above with flying colors. So far, it is experimentally correct, in the sense that string theories which closely imitate nature can be constructed. Equal in significance, it has been offering depth and elegance that are perhaps unprecedented, fusing into its set of notions many mathematical fields in a nontrivial fashion and giving rise to almost miraculous interrelations among these.

The subject of this paper is $N = 2$ string theory, first constructed in ref. [1] which in its structure appears to embody the aforementioned properties (for a review, see, ref. [2]). Its origin is in the pursual of semi-realism in the framework of non-trivial string theory. The requirements of supersymmetry and sufficiently large gauge group were shown to have a canonical solution. Namely, for any