Commun. Math. Phys. 140, 399–414 (1991)

Communications in Mathematical Physics

Quantum Group A_∞

Serge Levendorskiĭ[1] **and Yan Soibelman**[2]

[1] Rostov Institute for National Economy, SU-344798 Rostov-on-Don, USSR
[2] Rostov State University, SU-344006 Rostov-on-Don, USSR

Received January 15, 1991

Abstract. The quantum groups gl_∞ and A_∞ are construted. The representation theory of these algebras is developed and the universal R-matrix is presented.

0.1. The Lie algebra gl_∞ and its extension A_∞ play an important role in the theory of nonlinear equations [DJKM]. They are of interest as an example of Kac-Moody-Lie algebras of infinite type [K, FF]. Therefore it is natural to ask: what are the quantum analogues of these algebras in the sense of the quantum groups theory of Drinfeld [D1]? The answer is trivial for $gl_\infty = \varinjlim_n gl_n$, but this is not the case for A_∞. Some non-triviality is due tot the fact that there is no Lie algebra gl_∞ in the quantum group case [we have the quantized universal enveloping algebra $U_h(gl_\infty)$ only]. Hence one must analyse the completion of gl_∞ and the central extension of the corresponding algebra $\overline{gl}_\infty$ in terms of $U_h(gl_\infty)$ only. Moreover we need the Hopf Algebra structure in $U_h(A_\infty)$. This is essential in the case $h = 0$ already, because, for example, the well-known KP hierarchy is related to the equations for the orbit of highest vector in $L(\Lambda_0) \otimes L(\Lambda_0)$ where $L(\Lambda_0)$ is the basic representation of A_∞ [K, Chap. 14]. For the same reason we want to obtain $U_h(A_\infty)$ as the quasitriangular topological Hopf algebra [D1].

The purpose of the paper is to construct $U_h(gl_\infty)$ and $U_h(A_\infty)$ as quasi-triangular topological Hopf algebras and investigate the representation theory of these algebras. Some results along this lines have been obtained by Hayashi in [H]. Note that there are no constructions of $U_h(gl_\infty)$ and $U_h(A_\infty)$ as quantum groups in his paper.

0.2. Let us describe the contents. In Sect. 1 we construct the Hopf algebra $U_h(gl_\infty)$. This is the quantum analogue of gl_∞. The representations of $U_h(gl_\infty)$ in the spaces of sequences and (quantum) semi-infinite forms are given in Sect. 2. The Hopf algebra $U_h(A_\infty)$ (and some related algebras) is constructed in Sect. 3. This construction is more complicated than in the non-quantum case [K]. The representation theory of $U_h(A_\infty)$ is presented in Sect. 4. Our class