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An Algebraic Geometry Study of the b–c System with Arbitrary Twist Fields and Arbitrary Statistics

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Abstract. We present an analysis of the general b–c system (including the β–γ system) on a compact Riemann surface of arbitrary genus $g \geqq 0$ by postulating that its correlation functions should only have the singularities imposed by the operator product expansion (OPE) of the system. Studying a very (in fact optimally) general form of the b–c system, we prove rigorously that the standard practice of eliminating zero modes, and even the standard lagrangian, follow from the analyticity structure dictated by the OPE alone. We extend the analysis to consider the most general case of the presence of twist (e.g. spin) fields. We then determine all the possible correlation functions of the b–c system, with statistics unspecified, compatible with the OPE. On imposing Fermi and Bose statistics, we obtain the correlation functions of the fermionic b–c and β–γ systems, respectively.

1. Introduction

We consider a system consisting of a pair of quantum fields b, c on a compact, connected Riemann surface M of genus $g \geqq 0$. Our aim is to study the *correlation functions*, written symbolically as

$$C(m,n) \equiv \langle b(Q_1) \dots b(Q_m) c(P_1) \dots c(P_n) \rangle , \tag{1.1}$$

where the Q's and P's are arbitrary (but distinct) points on M, by postulating the *operator product expansion* (OPE)

$$b(Q)c(P) \sim \frac{I}{Q-P} . \tag{1.2}$$

Here $Q, P \in M$ while I, on the right-hand side of (1.2), is the *identity operator*. Equation (1.2) is to be understood as holding inside a correlation function $C(m,n)$. An immediate consequence of (1.2) is that a general correlation function $C(m,n)$ has a pole when the arguments of a b field and a c field coincide. We shall study the