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Stark Ladder Resonances for Small Electric Fields

J.-M. Combes* and P. D. Hislop*,,*****

Centre de Physique Théorique, CNRS, Luminy Case 907, F-13288 Marseille Cedex 9, France

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Abstract. We prove the existence of resonances in the semi-classical regime of small h for Stark ladder Hamiltonians $H(h,F) \equiv -h^2 \frac{d^2}{dx^2} + v + Fx$ in one-dimension. The potential v is a real periodic function with period τ which is the restriction to $\mathbb{R}$ of a function analytic in a strip about $\mathbb{R}$. The electric field strength F satisfies the bounds $\|v'\|_\infty > F > 0$. In general, the imaginary part of the resonances are bounded above by $ce^{-\kappa\rho_T h^{-1}}$, for some $0 < \kappa \leqq 1$, where $\rho_T h^{-1}$ is the single barrier tunneling distance in the Agmon metric for $v + Fx$. In the regime where the distance between resonant wells is $\mathcal{O}(F^{-1})$, we prove that there is at least one resonance whose width is bounded above by $ce^{-\alpha/F}$, for some $\alpha, c > 0$ independent of h and F for h sufficiently small. This is an extension of the Oppenheimer formula for the Stark effect to the case of periodic potentials.

1. Introduction

The Hamiltonian for an electron moving under the influence of a periodic potential and a constant electric field of strength $F \geqq 0$ in one-dimension is

$$H(h,F) = -h^2 \frac{d^2}{dx^2} + v + Fx. \tag{1.1}$$

The real periodic potential v is assumed to be the restriction to $\mathbb{R}$ of a function analytic in a strip about the real axis. We consider the small electric field regime $\|v'\|_\infty > F > 0$ in the semi-classical limit. We prove under these conditions that

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* Also PHYMAT, Départment de Mathématiques, Université de Toulon et du Var, F-83130 La Garde, France

** Permanent address: Mathematics Department, University of Kentucky, Lexington, KY 40506-0027, USA

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