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Transversal Dirac Families in Riemannian Foliations

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Abstract. We describe a family of differential operators parametrized by the transversal vector potentials of a Riemannian foliation relative to the Clifford algebra of the foliation. This family is non-elliptic but in certain ways behaves like a standard Dirac family in the absolute case as a result of its elliptic-like regularity properties. The analytic and topological indices of this family are defined as elements of K-theory in the parameter space. We indicate how the cohomology of the parameter space is described via suitable maps to Fredholm operators. We outline the proof of a theorem of Vafa–Witten type on uniform bounds for the eigenvalues of this family using a spectral flow argument. A determinant operator is also defined with the appropriate zeta function regularization dependent on the codimension of the foliation. With respect to a generalized coupled Dirac–Yang–Mills system, we indicate how chiral anomalies are located relative to the foliation.

1. Introduction

This paper provides a setting for the study of a coupled Dirac–Yang–Mills theory in the presence of a Riemannian foliation $\mathscr{F}$. The latter may be regarded in essence as a generalized dynamical system in which a one-dimensional foliation is simply called a flow. Among a number of possible applications suggested in this paper, we mention at this stage a particular example which is well known and which may serve as a partial motivation for what follows: The pure Yang–Mills equations on $\mathbb{R}^4$ may be dimensionally reduced by requiring translation invariance in one direction, thus arriving at a Higgs system in $\mathbb{R}^3$ yielding magnetic monopole equations [A–H][J–T]. One may see the generalization of translation invariance in the above example as invariance under the flows in the leaf direction of a foliation $\mathscr{F}$. In the context of vector potentials this leads to the notion of a *basic* connection

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