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Localization for Random Schrödinger Operators with Correlated Potentials

Henrique von Dreifus[1*,**] **and Abel Klein**[2***]

[1] Department of Physics, Princeton University, Princeton, NJ 08544, USA
[2] Department of Mathematics, University of California at Irvine, Irvine, CA 92717, USA

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Abstract. We prove localization at high disorder or low energy for lattice Schrödinger operators with random potentials whose values at different lattice sites are correlated over large distances. The class of admissible random potentials for our multiscale analysis includes potentials with a stationary Gaussian distribution whose covariance function $C(x,y)$ decays as $|x-y|^{-\theta}$, where $\theta>0$ can be arbitrarily small, and potentials whose probability distribution is a completely analytical Gibbs measure. The result for Gaussian potentials depends on a multivariable form of Nelson's best possible hypercontractive estimate.

1. Introduction

We consider the random Schrödinger operator $H=-\Delta+V$ on $l^2(\mathbf{Z}^d)$, where Δ is the centered finite difference Laplacian, i.e., $\Delta(x,y)=1$ if $|x-y|=1$ and zero otherwise, and V is an ergodic potential, i.e., $\{V(x);\, x\in\mathbf{Z}^d\}$ is an ergodic stochastic process. The motivation for studying this class of operators comes from Solid State Physics, where one is interested in the behavior of an electron in a random background. This model was first introduced by Anderson [1] and is known as the Anderson tight-binding model.

It is well known that the spectrum of the random operator H is independent of the choice of potential with probability one [2, 3, 4]. The same is true of the decomposition of the spectrum into pure point, absolutely continuous and singular continuous spectrum [3, 4].

The random operator H exhibits localization in an energy interval I if it has only pure point spectrum in I with probability one. In this case, if the eigenfunc-

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* Current address: Instituto de Mathematica e Estatistica, Universidade de São Paulo, São Paulo, S.P., Brazil

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