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Exchange Algebra and the Drinfeld–Sokolov Theorem

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Abstract. We analyse the relation between the exchange algebra and the separation of the chiralities in classical Toda field theory. We show that there exists a conformally covariant Bloch wave basis such that the two chiralities commute. In terms of this basis we then reconstruct the periodic and local solution of Toda field theory.

1. Introduction

Toda field theories are simultaneously conformal field theories and integrable models. A basic tool in the study of these theories is the exchange algebra [1, 2]. Since we have a conformal theory, we expect the dynamics to split into a right moving sector and a left moving sector. However it so happens that the simplest choice for the exchange algebra coming from the integrable structure does not completely split the chiralities due to the zero mode problem. The aim of this paper is to analyse the relation between the exchange algebra and the separation of the chiralities in classical Toda field theory. In this first section we illustrate the problem more precisely, introduce the notation and outline the solution. The rest of the paper is devoted to the proofs of the various propositions.

Let $\mathcal{G}$ be a simple finite dimensional Lie algebra of rank n, equipped with an invariant scalar product denoted by $(,)$. We choose a Cartan subalgebra $\mathcal{H}$ with an orthonormal basis $\{H_i\}$. We also need the Cartan decomposition of $\mathcal{G}$,

$$\mathcal{G} = \mathcal{N}_- \oplus \mathcal{H} \oplus \mathcal{N}_+ .$$

We recall the following commutation relations in a Cartan–Weyl basis:

$$[H, E_{\pm\alpha}] = \pm\alpha(H) E_{\pm\alpha},$$
$$[E_\alpha, E_{-\alpha}] = H_\alpha .$$

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