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Flat Periodic Representations of $\mathscr{U}_q(\mathscr{G})$

D. Arnaudon and A. Chakrabarti

Centre de Physique Théorique de l'Ecole Polytechnique, F-91128 Palaiseau Cedex, France, UPR 14-CNRS

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Abstract. We give explicit expression of flat periodic representations, when they exist, of the quantum analogues of simple Lie algebras and their affine extensions for a parameter of deformation q equal to a root of unity. By "flat periodic," we mean that these representations have no highest weight and that all the weights have multiplicity 1.

1. Introduction

In [1] and [2], it was proved that $\mathscr{U}_q(SU(2))$ have periodic representations (i.e. with injective action of generators) parametrized by 3 continuous parameters. In [3], this was extended to $\mathscr{U}_q(SU(3))$ and in [4, 5] to $\mathscr{U}_q(SU(n))$. In [6], De Concini and Kac studied the representations of $\mathscr{U}_q(\mathscr{G})$ at root of unity. They proved that their dimensions are bounded and that they were parametrized by $\dim \mathscr{G}$ complex continuous parameters. (We prefer the word periodic instead of cyclic, since cyclic has already its own meaning in the theory of modules.)

Periodic representations of $\mathscr{U}_q(SU(2))$ have proved their interest in the generalization of the chiral Potts model [7, 8]. Periodic representations of $\mathscr{U}_q(SU(3))$ are used for the same purpose in [9, 10], and this is extended to flat periodic representations of $\mathscr{U}_q(A_n^{(1)})$ in [11]. In [12], (flat) periodic representations of $\mathscr{U}_q(A_n^{(2)})$ and their intertwiners are related to the Boltzman weights of another statistical model, i.e. the Izergin-Korepin model.

In this paper, we consider "flat" periodic representations, i.e. for which all the weight spaces have dimension 1. If $q^m = 1$, the dimension of flat periodic representations of $\mathscr{U}_q(\mathscr{G})$ is $m^{\operatorname{rank} \mathscr{G}}$. Flat periodic representations are minimal periodic representations, in the sense that when they exist, their dimensions are the smallest possible. We will in the following give an explicit expression of flat periodic representations of $\mathscr{U}_q(\mathscr{G})$, for $\mathscr{G}$ a simple Lie algebra or an affine extended Lie algebra. We prove in fact that there is no flat periodic representation if the Dynkin diagram has a branching point, or a triple link or an exten-