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1D-Quasiperiodic Operators. Latent Symmetries

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Abstract. A large class of discrete quasiperiodic operators is shown to be decomposed into orbits of $SL(2,\mathbb{Z})$ action with equal densities of states. Moreover under some natural assumptions all nontrivial representatives of the mentioned action transform operators with pure point spectrum into those with absolutely continuous spectrum. Some applications of these results are presented.

1. Introduction

Let us consider the general family of quasiperiodic operators acting in $l^2(\mathbb{Z})$ through the formula

$$(H\Psi)_l = \sum_{j=-\infty}^{\infty} \Psi_{l-j} f_j(\Phi + l\alpha), \quad f_j \in C(S^1),\ \alpha \in \mathbb{R}\backslash\mathbb{Q}. \tag{1}$$

This paper is an attempt of outlook on this class as a whole in order to reveal which almost periodic features are responsible for various spectral properties and thus to provide some old results with a new understanding.

During the 80's there was a great amount of interest in almost periodic operators. The balance between generality and concreteness was periodically driven to either side. Perhaps the most attention was paid to investigations of the almost-Mathieu operator

$$(H\Psi)_l = \Psi_{l+1} + \Psi_{l-1} + \lambda \cos(\alpha l + \Phi)\Psi_l \tag{2}$$

and related models.

The almost-Mathieu equation arose from the model of Bloch electron in the uniform magnetic field [1] and has been studied extensively both from physical and mathematical points. It seemed to have very interesting properties but (in comparison with a related 2D-operator) a simple easy to handle form.

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