Commun. Math. Phys. 139, 395–432 (1991)

Communications in
Mathematical
Physics

Proof of Chiral Symmetry Breaking in Strongly Coupled Lattice Gauge Theory

Manfred Salmhofer * **and Erhard Seiler**

Max-Planck-Institut für Physik und Astrophysik, Werner-Heisenberg Institut,
Föhringer Ring 6, W-8000 München 40, Federal Republic of Germany

Received January 8, 1991

Abstract. We study chiral symmetry in the strong coupling limit of lattice gauge theory with staggered fermions and show rigorously that chiral symmetry is broken spontaneously in massless QED and the gauge-invariant Nambu-Jona-Lasinio model if the dimension of spacetime is at least four. The results for the chiral condensate as a function of the mass imply that the mean-field approximation is an upper bound for this observable which becomes exact as the dimension goes to infinity. For the model with gauge group $U(N)$, $N = 2, 3, 4$, we prove that chiral long-range order exists at zero mass in four or more dimensions.

1. Introduction

The idea that the pion is an approximate Nambu-Goldstone particle induced by the (approximately) spontaneous breaking of the axial $SU(2)$ invariance, and that the same mechanism gives most of its mass to the nucleon, is by now thirty years old [1, 2]. It has survived the tremendous changes that have occurred since then in the description of strong interactions and is nowadays generally believed to be realized in QCD. Nevertheless to this day a more rigorous mathematical understanding of this phenomenon, going beyond the mean-field analysis in [2] is lacking.

Heuristic arguments that were advanced much more recently suggested that spontaneous breaking of chiral symmetry should be in some way related to confinement in QCD with massless quarks [3]; these arguments still hinge on the relation of chiral symmetry breaking and condensation phenomena that was for purely fermionic theories first analyzed in the above-mentioned pioneering papers by Nambu and Jona-Lasinio [1, 2]. In their treatment the

* Address after August 1991: Mathematics Department, University of British Columbia, Vancouver, Canada V6T1Y4