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Holonomy Groups, Complex Structures and $D = 4$ Topological Yang-Mills Theory

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Abstract. We study conditions for the existence of extended supersymmetry in topological Yang-Mills theory. These conditions are most conveniently formulated in terms of the holonomy group of the underlying manifold, on which the topological Yang-Mills theory is defined. For irreducible manifolds we find that extended supersymmetries are in 1-1 correspondence with covariantly constant complex structures. Therefore, the topological Yang-Mills theory on any Kähler manifold possesses one additional supersymmetry and on any hyperKähler manifold there are three additional supersymmetries. The Donaldson map, which plays a crucial role in the construction of the topological invariants, is generalized for Kähler manifolds, thus providing candidates for new invariants of complex manifolds.

1. Introduction

Recently, a quantum field theory method was proposed for constructing topological invariants of four dimensional manifolds [1]. It is based on $N = 2$, $D = 4$ Euclidean supersymmetric Yang-Mills theory minimally coupled to gravity. One might expect that the $N = 2$ supersymmetry of the theory is completely broken because the graviton's superpartners are absent. Nevertheless it turns out that the theory is invariant under a singlet rigid supersymmetry if one identifies the $SU(2)$ automorphism group of $N = 2$ supersymmetry with the $SU(2)_L$ subgroup of the Euclidean tangent group $SO(4) \simeq SU(2)_L \times SU(2)_R$. This singlet supersymmetry plays an important role in the construc-

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