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Opacity of an Automaton. Application to the Inhomogeneous Ising Chain

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Abstract. An automaton maps infinite sequences onto infinite sequences. We define the opacity as the distance between output sequences and input sequences. A transparent automaton hardly disturbs the input sequence. An opaque automaton erases some of the information contained in the input sequence. We apply these ideas to the study of the inhomogeneous Ising chain governed by the Hamiltonian

$$\mathscr{H}_\varepsilon(\sigma) = -J \sum_{q=-M}^{N-1} \varepsilon_q \sigma_q \sigma_{q+1} - H \sum_{q=-M}^{N} \sigma_q .$$

Part One. Automata

I.1. Definition of an Automaton

Problems in one-dimension physics often depend on the solution of a recurrence relation of the type

$$\delta_{n+1} = f(\delta_n),$$

where f is a given map and where δ_0 is known. It may happen that the function f itself varies from step to step. For example, suppose we are given two functions f_+ and f_- and a sequence of signs (ε_n), $\varepsilon_n \in \{-, +\}$. Then the sequence (δ_n) defined by

$$\delta_{n+1} = f_{\varepsilon_n}(\delta_n)$$

depends on the sequence (ε_n). This situation arises in the study of the inhomogeneous Ising Chain where the coupling constant varies from one site to the other (binary alloy, chain with impurities, spin glass, ...).

The object of the paper is to give a general discussion of the map $(\varepsilon_n) \mapsto (\delta_n)$ and this can be achieved in terms of automata theory. We then apply our results to the Ising Chain.