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Intersection Properties of Invariant Manifolds in Certain Twist Maps

J. J. P. Veerman[1] **and F. M. Tangerman**[2]

[1] Institute for the Mathematical Sciences, SUNY at Stony Brook, Stony Brook, NY 11794, USA
[2] Mathematics Department, SUNY at Stony Brook, Stony Brook, NY 11794, USA

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Abstract. We consider the space N of C^2 twist maps that satisfy the following requirements. The action is the sum of a purely quadratic term and a periodic potential times a constant k (hereafter called the nonlinearity). The potential restricted to the unit circle is bimodal, i.e. has one local minimum and one local maximum. The following statements are proven for maps in N with nonlinearity k large enough. The intersection of the unstable and stable invariant manifolds to the hyperbolic minimizing periodic points contains minimizing homoclinic points. Consider two finite pieces of these manifolds that connect two adjacent homoclinic minimizing points (hereafter called fundamental domains). We prove that all such fundamental domains have precisely one point in their intersection (the Single Intersection theorem).

In addition, we show that limit points of minimizing points are recurrent, which implies that Aubry Mather sets (with irrational rotation number) are contained in diamonds formed by local stable and unstable manifolds of nearby minimizing periodic orbits (the Diamond Configuration theorem). Another corollary concerns the intersection of the minimax orbits with certain symmetry lines of the map.

I. Introduction and Results

The main objective of this work is to bound the number of ways that stable and unstable manifolds of minimizing orbits can intersect each other. We do this for a class of maps whose members are close (in the C^2-topology) to a standard map with large k.

We will consider maps generated by the action:

$$h(x,x')=1/2(x-x')^2+kV(x),$$

where

$$V\in C^3(S^1),$$