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On the Chern Character of θ Summable Fredholm Modules

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Abstract. We show that the entire cyclic cohomology class given by the Jaffe-Lesniewski-Osterwalder formula is the same as the class we had constructed earlier as the Chern character of θ-summable Fredholm modules.

1. Introduction

Cyclic cohomology replaces de Rham homology in the set up of non-commutative differential geometry ([1, 2]). In particular it is a natural receptacle for the Chern character in K-homology ([1]) so that to each K homology cycle of finite dimension, on an algebra A, there corresponds a stable cyclic cohomology class. This class reduces to the index class ([1, 2]) for the K-homology cycle associated to an elliptic differential operator on a manifold M, (where $A = C^\infty(M)$ is the algebra of smooth functions on M). One of the distinctive features of cyclic cohomology is that it fits naturally not only with the non-commutative case but also with the infinite dimensional situation. Indeed, stable (or periodised) cyclic cohomology is the cohomology of cochains with finite support in the (b, B) bicomplex of the algebra A ([1]) and by imposing a suitable growth condition on cochains with infinite support, we introduced in [3] the cohomology of A, which is relevant for the infinite dimensional situation.

In particular it allows to extend the Chern character in K-homology to K-homology cycles $(\mathscr{h}, D)$ on the algebra A (cf. [3]), where the operator D is no longer **finitely summable** (i.e. $\mathrm{Tr}(D^{-p}) < \infty$ for some $p < \infty$) but is only θ-**summable**: $\mathrm{Tr}(e^{-\beta D^2}) < \infty$. Our original construction ([3]) of this Chern character was based on the correspondence between cocycles with infinite support and traces on the algebras QA, εA of Cuntz and Zekri [5, 9]. The algebra εA is an essential ideal in the free product $A * \mathbb{C}(\mathbb{Z}/2)$ of A by the group ring of the group $\mathbb{Z}/2\mathbb{Z}$. The growth condition of **entire** cocycles corresponds to the **vanishing** of the spectral radius of all elements of εA for the trace given by the cocycle. Thus any homomorphism